Model of joint displacement using sigmoid function

Experimental approach for planar pointing task and squat jump

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Content

Abstract

Using an experimental based optimization approach, this study investigated whether two specific human movements, i.e. pointing tasks and squat-jumps, can be modeled using a reduced set of parameters. Three models were proposed to model the evolution of joint angles as sigmoid shaped curves. 304 pointing tasks and 120 squat-jumps performed by 22 and 13 subjects respectively were used to quantify the accuracy of the models. The parameters of the sigmoids were optimized to achieve the best fitting of experimental data, i.e. the 2D position of the joints throughout the movements. The results showed that the models were able to reproduce accurately both movements. This study provides a new framework to model planar movements with a small number of meaningful kinematic parameters, allowing a continuous description of the kinematics and kinetics of the tasks.

These results come out from paper available on arXiv [Cre+12].

Content

Objectifs and Assumptions

- Planar polyarticulated system;
- Rigid segments;
- The first point is considered to be immobile;

$\Longrightarrow$ Joints coordinates are determinated according to the joint angles only.

- Replace the experimental trajectories by modelized analytical curves.
- The trajectories become smooth and can be analytically differentiated or used for theoretical calculus.

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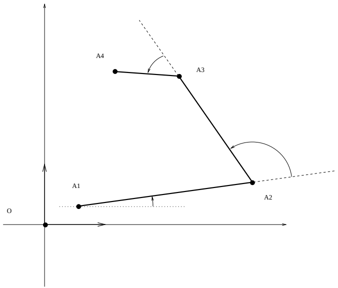
Open chain description

- Chain defined by points A_1, ..., A_p where the origin A_1 is fixed.
- Knowns parameters $l_j = A_j A_{j+1}$.
- The coordinates of A_j $(1 \leq j \leq p)$ can be determined according to the joint angles θ_k $(1 \leq k \leq p-1)$.



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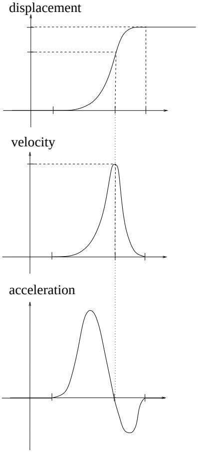
principle

- Each joint angle time history is described by a "sigmoid" form.
- Begin and end of movements occur at t_b and t_e.
- The velocity is maximal at $t_0 \in (t_b, t_e)$.



problem

We seek a function g of class C^2 defined on $[0,1]$ satisfying:

$$g(0)=0, \quad g'(0)=0, \quad g''(0)=0, \tag{1a}$$

$$g(1)=1, \quad g'(1)=0, \quad g''(1)=0, \tag{1b}$$

$$g(\alpha)=\beta, \quad g'(\alpha)=k, \quad g''(\alpha)=0, \tag{1c}$$

$$\forall u \in (0,\alpha), \quad g''(t)>0, \tag{1d}$$

$$\forall u \in (\alpha,1), \quad g''(t)<0. \tag{1e}$$

Classical sigmoid

A lot of sigmoid models in statistics or mechanics.
See for example, the log-normal function used by Plamodon [Pla98; Pla95a; Pla95b; PCF03].
However, there is not general construction for problem (1).
Each built sigmoid is defined by 7 parameters. So, if the coordinates of the studied movement are defined by $p-1$ joints angles, we need $7(p-1)$ parameters to describe the whole movement.

Solution

Three families of models (see details in [Cre+12]):

- The first one (SYM model) is of class C^2 but not C^3 and expressed as an exponential function: $H^{(a,b,\kappa)}(u) = a\left(1 - e^{-bu^{\kappa}}\right)$, defined on $[0,\alpha]$ and deduiced from this function on $[\alpha, 1]$.
- The second one (NORM model) is of class C^∞ and expressed with the erf function defined by $\forall x \in \mathbb{R}, \quad \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$, and by using a bijection from $(0,1)$ to $\mathbb{R}$.
- The third one (INVEX model) is also of class C^∞ and uses a function defined on $[0,1]$ according to $t \mapsto \exp\left(-\frac{1}{t^\lambda (1-t)^\mu}\right)$ and its integral.

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Pointing tasks and squat-jumps

9 right-handed male subjects were asked to perform pointing tasks in the horizontal plane. The total number of pointing tasks was 304.

The studied joints are the shoulder, elbow, wrist and forefinger extremity ($p = 4$).

The squat-jump data was obtained from a previous work [BBM13]. Each of 13 other subjects performed 10 vertical jumps. Instructions were given for keeping the hands on the hips during the movement to limit the contribution of the upper limbs to the performance. The studied joints are the shoulder, the hip, the knee, the ankle and the toe ($p = 5$).

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Data processing

First optimization consisted in minimizing the sum of square of differences between experimental angles θ^i and those obtained from the sigmoid models σ^i for each of n instants:

$$\mathcal{S} = \sum_{i=1}^{n} \left(\theta^i - \sigma^i\right)^2.$$

Secondly, differences between experimental and model reconstructed joint positions were minimized in a least square sense. The objective can thus be written as

$$\mathcal{S}' = \sum_{j=1}^{p} \sum_{i=1}^{n} \left(X_j^i - x_j^i\right)^2 + \left(Y_j^i - y_j^i\right)^2.$$

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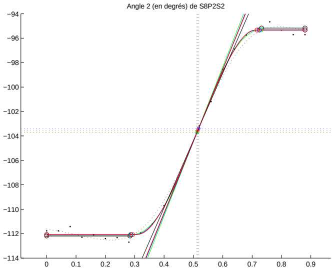
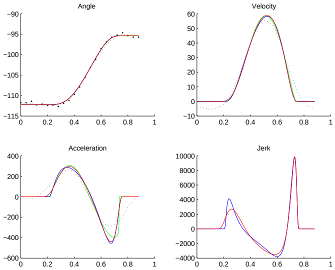
Example for the second joint angle for pointing task

Results of the first optimization

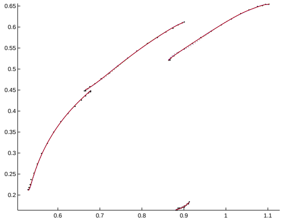


Results of the second optimization



Experimental and calculated trajectories

- Experimental data: black points;
- Smoothing data: dashed black line;
- INVEXP: red continuous line;
- NORM: blue continuous line;
- SYM: green continuous line.



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Numerical viewpoint

The obtained results are very accurate from numerical viewpoint. Indeed, for pointing task, in 95% of cases, for the three models, the maximal error *i.e.* the difference between the calculated trajectories and the experimental trajectories is smaller than

$$\varepsilon_{\text{max}} = 1.86 \text{ cm}.$$

The mean error is smaller than

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For squat jumps, the maximal error is smaller than

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Final animations

1. for pointing task:
    - Click here to see experimental movie
    - Movies built with sigmoid models
        - Click here for SYM model
        - Click here for NORM model
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    - Click here to see experimental and the three sigmoid models
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[BBM13] J. Bastien, Y. Blache, and K. Monteil. *Estimation of anthropometrical and inertial body parameters using double integration of residual torques and forces during squat jump*. 2013. arXiv:1305.6426.

[Cre+12] T. Creveaux, J. Bastien, C. Villars, and P. Legreneur. *Model of joint displacement using sigmoid function. Experimental approach for planar pointing task and squat jump*. 2012. arXiv:1207.2627.

[PCF03] R. Plamondon and A. Chunhua Feng. “A kinematic theory of rapid human movements. Part IV. a formal mathematical proof and new insight”. In: *Biol. Cybern.* 89 (2003), pages 126–138. DOI: 10.1007/s00422-003-0407-9.

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[Pla95b] R. Plamondon. “A kinematic theory of rapid human movements. Part II. Movement time and control”. In: *Biol. Cybern.* 72.4 (1995), pages 309–320. DOI: 10.1007/BF00202786.

[Pla98] R. Plamondon. “A kinematic theory of rapid human movements. Part III. Kinetic outcomes”. In: *Biol. Cybern.* 78 (1998), pages 133–145.